S–2355

Sub. Code
22MMA1C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

ALGEBRA — I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a normal subgroup. Give an example.

2. Define an even permutation. Give an example.

3. State second part of Sylow's theorem.

4. Define an internal direct product.

5. Define a division ring. Give an example.

6. When will you say that two rings are said to be homomorphism?

7. Define an ideal of a ring. Give an example.

8. Write short notes on embedded ring and over ring.

9. Define relatively prime element.

10. Define primitive polynomial and content of a polynomial.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H.

Or

(b) State and prove the Cauchy's theorem for abelian groups.

12. (a) State and prove third part of Sylow's theorem.

Or

(b) Let G be a group and suppose that G is the internal direct product of $N_1, N_2, ..., N_n$. Let $T = N_1 \times N_2 \times ..... \times N_n$. Prove that G and T are isomorphic.

13. (a) If R is a ring, then prove that for all $a, b \in R$

(i) $a0 = 0a = 0$;

(ii) $a(-b) = (-a)b = -(ab)$;

(iii) $(-a)(-b) = ab$.

If, in addition, R has a unit element 1, then prove that

(iv) $(-1)a = -a$;

(v) $(-1)(-1) = 1$.

Or

(b) Prove that the homomorphism ϕ of R into R' is an isomorphism if and only if $I(\phi) = (0)$.

14. (a) If R is a commutative ring and $a \in R$,

(i) Show that $a\,R = \{a\,r \,/\, r \in R\}$ is a two-sided ideal of R.

(ii) Show by an example that this may be false if R is not commutative.

Or

(b) If $[a,b] = [a',b']$ and $[c,d] = [c',d']$ then prove that $[a,b]\,[c,d] = [a',b']\,[c',d']$.

15. (a) Define a Euclidean ring. Also prove that a Euclidean ring possesses a unit element.

Or

(b) State and prove the division algorithm for polynomials.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove the Cayley's theorem.

17. (a) With the usual notations, prove that $O\,(G) = \sum \frac{O\,(G)}{O\,(N(a))}$, where this sum runs over one element a in each conjugate class.

(b) If p is a prime number and $p \,/\, O(G)$, then prove that G has an element of order p.

18. Suppose $C = \{(\alpha, \beta) \mid \alpha, \beta \in \mathbb{R}$ define $(\alpha, \beta) = (\gamma, \delta) \Leftrightarrow \alpha = \gamma$ and $\beta = \delta$. Prove that C is a field under the suitable binary operations in C.

19. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R / M is a field.

20. (a) State and prove the Gauss' Lemma.

(b) Prove that the polynomial $1 + x + ... + x^{p-1}$, where p is a prime number, is irreducible over the field of rational numbers.

S–2356

Sub. Code
22MMA1C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

ANALYSIS – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A (10 × 2 = 20)

Answer **all** questions.

1. Define Open cover.

2. What do you mean by connected metric space?

3. State Ratio Test.

4. Prove that $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

5. What do you mean by absolutely convergences?

6. State Abel's theorem.

7. What do you mean by uniformly continuous function?

8. Define monotonically decreasing function.

9. State Chain Rule for differentiation.

10. Define local minimum at a point.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) If X is a metric space and $E \subset X$, then prove that

(i) $\overline{E}$ is closed.

(ii) $E = \overline{E}$ if and only if E is closed.

(iii) $\overline{E} \subset F$ for every closed set $F \subset X$ such that $E \subset F$.

Or

(b) If $\{K\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K\alpha\}$ is nonempty. Prove that $\bigcap K\alpha \neq \phi$.

12. (a) Suppose $\{S_n\}, \{t_n\}$ are complex sequences and $\lim_{n\to\infty} S_n = S$, $\lim_{n\to\infty} t_n = t$. Prove that $\lim_{n\to\infty} \frac{1}{S_n} = \frac{1}{S}$, provided $S_n \neq 0, n = 1, 2....$ and $S \neq 0$.

Or

(b) Prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

13. (a) If $\sum an$ is a series of complex numbers which converges absolutely then prove that every rearrangement of $\sum an$ converges and they all converge to the same sum.

Or

(b) Suppose the radius of convergence of $\sum c_n z^n$ is 1, and suppose $c_o \geq c_1 \geq c_2 \geq ....$ $\lim_{n\to\infty} c_n = 0$. Prove that $\sum c_n z^n$ converges at every point on the circle $|z| = 1$, except possibly at $z = 1$.

14. (a) State and prove Intermediate value theorem.

Or

(b) Prove that a continuous function of a continuous function is continuous.

15. (a) If f and g are continuous real functions on $[a, b]$ which are differentiable in (a, b), prove that there is a point $x \in (a, b)$ at which $[f(b) - f(a)]g'(x) = [g(b) - g(a)]\, f'(x)$.

Or

(b) Suppose f is a continuous mapping of $[a, b]$ into R^k and f is differentiable in (a, b), prove that there exists $x \in (a, b)$ such that $|f(b) - f(a)| \leq (b - a)|f'(x)|$

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove Weierstrass theorem.

17. State and prove Root test.

18. Suppose (a) $|c_1| \geq |c_2| \geq |c_3| \geq .....$

(b) $c_{2m-1} \geq 0,\ c_{2m} \geq 0,\ m = 1, 2....$

(c) $\lim_{n \to \infty} c_n = 0$.

Prove that $\sum c_n$ converges.

19. Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

20. State and prove Taylor's theorem.

S–2357

Sub. Code
22MMA1C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

DIFFERENTIAL GEOMETRY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Write down the equation of the fundamental planes.

2. When will you say that curves are said to be Bertrand curves?

3. Write a short note on the general surface of revolution.

4. Write down the first fundamental form of the surface.

5. Show that at every point of a geodesic the rectifying plane is tangent to the surface.

6. State the existence theorem for geodesic.

7. What is meant by geodesic curvature vector of a curve on a surface?

8. Prove that a curve on a surface is geodesic if and only if geodesic curvature vector is zero.

9. Define the Gaussian curvature.

10. Write down the characteristic line corresponding to the plane.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Show that involutes of a circular helix are plane curves.

Or

(b) Show that a necessary and sufficient condition that a curve be a helix is that $\left[\vec{r}^{\,II}, \vec{r}^{\,III}, \vec{r}^{\,IV}\right] = -k^5 \frac{d}{ds}\left(\frac{\tau}{k}\right) = 0$.

12. (a) Show that the metric is a positive definite quadratic form in du and dv.

Or

(b) If (l,m) are direction consines and (λ, μ) be the direction ratios, then prove that $(l,m) = \frac{(\lambda, \mu)}{\left(E\lambda^2 + 2F\lambda\mu + G\mu^2\right)^{\frac{1}{2}}}$.

13. (a) Define christoffel symbols and express it in terms of first derivatives of fundamental coefficients.

Or

(b) Prove that the curves of the family $\frac{v^2}{u^2}$ = constant are geodesics on a surface with metric $v^2\, du^2 - 2uv\, du\, dv + 2u^2\, dv^2$; $u > 0, v > 0$.

14. (a) Prove that every helix on a cylinder is a geodesic and conversely geodesic on any cylinder is helix.

Or

(b) Prove that $k_g = \frac{1}{H\, i^3}\left(V\frac{\partial T}{\partial \dot{u}} - U\frac{\partial T}{\partial \dot{v}}\right)$.

15. (a) If L, M, N Vanish everywhere on a surface, then prove that the surface is part of a plane.

Or

(b) Show that the principal radii of curvature of the surface $y\cos\left(\frac{z}{a}\right) = x\sin\left(\frac{z}{a}\right)$ are equal to $\pm\frac{\left(x^2 + y^2 + z^2\right)}{a}$. Find the lines of curvature.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. State and prove the fundamental existence theorem for space curves.

17. If Q is the angle at the point (u, v) between the two directions given by $P\, du^2 + 2\, Q\, du\, dv + R\, dv^2 = 0$, then prove that $\tan\theta = \frac{2\, H\, (Q^2 - PR)^{1/2}}{ER - 2\, FQ + GP}$.

18. If the orthogonal trajectories of the curve V - constant are geodesic, then prove that $\frac{H^2}{E}$ is independent of u.

19. State and prove the Gauss - Bonnet theorem.

20. State and prove the Euler's theorem.

S–2358

Sub. Code
22MMA1C4

M.Sc. DEGREE EXAMINATION, APRIL 2024

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours | Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Determine whether the functions $\phi_1(x) = 1$, $\phi_2(x) = x$, $\phi_3(x) = x^3$, $x \in (-\infty, \infty)$ are linearly dependent or independent.

2. Write short note on the non-homogeneous equation of order n.

3. State the Hermite equation.

4. Prove that $P_n(-x) = (-1)^n P_n(x)$.

5. Define a regular singular point of the equation.

6. State the Laguerre equation.

7. Show that $x^{1/2} J_{-1/2}(x) = \frac{\sqrt{2}}{\Gamma(1/2)} \cos x$.

8. Solve $y' = 3y^{2/3}$.

9. Let $f(x, y) = 4x^2 + y^2$, on $S: |x| \leq 1$, $|y| \leq 1$. Show that f satisfies a Lipschitz condition on the set S.

10. State existence theorem for convergence of the successive approximations.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) State and prove uniqueness theorem for linear equation with constant coefficients $L(y) = 0$.

Or

(b) Find the solution ϕ of the initial value problem $y''' + y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

12. (a) One solution of $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) Show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$.

13. (a) Compute the indicial polynomials and their roots, for the equation $x^2 y'' + (\sin x)y' + (\cos x)y = 0$.

Or

(b) Find all solution ϕ of the form $\phi(x) = |x|^r \sum_{k=0}^{\infty} c_k x^k$, $(|x| > 0)$, for the equation $x^2 y'' + xy' + x^2 y = 0$.

14. (a) With the usual notations, prove that the series defining J_0 and K_0 converge for $|x| < \infty$.

Or

(b) Find an integrating factor for the following equation and solve them : $\cos x \cos y dx - 2\sin x \sin y dy = 0$.

15. (a) Show that all the successive approximations for the problem $y' = y^2$, $y(0) = 1$, exist for all real x.

Or

(b) Consider the initial value problem $y' = xy + y^{10}$, $y(0) = \frac{1}{10}$. Show that a solution ψ of this problem exists for $|x| \leq \frac{1}{2}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find all solutions of the equation $y'' - 2iy' - y = e^{ix} - 2e^{-ix}$.

17. Find two linearly independent power series solutions (in powers of x) of the equation $y'' + x^2 y' + x^2 y = 0$.

18. Let ϕ be a solution for $x > 0$ of the Euler equation $x^2 y'' + axy' + by = 0$, where a, b are constants. Let $\psi(t) = \phi(e^t)$.

(a) Show that ψ satisfies the equation $\psi''(t) + (a-1)\psi'(t) + b\psi(t) = 0$.

(b) Compute the characteristic polynomial of the equation satisfied by ψ and compare it with the indicial polynomial of the given Euler equation.

19. Derive Bessel function of order α of the first kind.

20. Let f be a real-valued continuous function on the strip $S : |x - x_0| \leq a$, $|y| < \infty$, $(a > 0)$ and suppose that f satisfies on S a Lipschitz condition with constant $k > 0$. Show that the successive approximations $\{\phi_k\}$ for the problem $y' = f(x, y)$, $y(x_0) = y_0$, ———————— (1) exist on the entire interval $|x - x_0| \leq a$ and converge there to a solution ϕ of (1).

S–2362

Sub. Code
22MMA2C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

ALGEBRA — II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define Homomorphism between the vector spaces.

2. Prove that $L(S)$ is a subspace of V.

3. Define a basis and dimension of a vector space over F.

4. Show that $W^{\perp}$ is a subspace of V over F.

5. Define an algebraic number.

6. For any $f(x), g(x) \in F[x]$, prove that $[f(x) + g(x)]' = f'(x) + g'(x)$.

7. Define invertible linear transformation in $A(V)$.

8. Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if for some $v \neq 0$ in V, $T(v) = \lambda v$.

9. What do you mean by unitary and Hermitian Linear transformation in $A(V)$?

10. If T is unitary and if λ is a characteristic root of $T \in A(V)$, then prove that $|\lambda| = 1$.

Part B $(5 \times 5 = 25)$

Answer **all** the questions, choosing either (a) or (b).

11. (a) If V is the internal direct sum of $U_1, U_2, ...U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, ...U_n$.

Or

(b) Prove that if $v, v_2, ...v_n$ is a basis of V over F and if $w_1, w_2, ...w_m$ in V are linearly independent over F, then $m \leq n$.

12. (a) Prove that $Hom(V,W)$ is a vector space over F under the suitable operations in $Hom(V,W)$.

Or

(b) If V is a finite dimensional inner product space and W is a subspace of V, then prove that $(W^{\perp})^{\perp} = W$.

13. (a) Prove that the elements in K which are algebraic over F form a sub field of K.

Or

(b) State and prove the Remainder theorem.

14. (a) Prove that if V is a finite dimensional vector space over F, then $T \in A(V)$ is invertible if and only if T maps V onto V.

Or

(b) Show that the eigen vectors corresponsing to distinct eigen values are linearly independent over F.

15. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

Or

(b) If $(T(v), T(v)) = (v, v)$ for all $v \in V$, then prove that T is unitary.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. If V is finite-dimensional and if W is a subspace of V, then prove that W is finite-dimensional, $\dim W \le \dim V$ and $\dim\left(\frac{V}{W}\right) = \dim(V) - \dim(W)$.

17. Establish the Gram-Schmidt orthogonalization process.

18. Prove that the element $a \in k$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

19. If V is finite-dimensional over F, then for $S, T \in A(V)$, prove the following

(a) $r(ST) \le r(T)$

(b) $r(TS) \le r(T)$

(c) $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$.

20. Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

S–2363

Sub. Code
22MMA2C2

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** the questions.

1. Define refinement and common refinement.

2. State the integration by parts theorem.

3. Define uniform convergence on a set E.

4. What do you mean by equicontinuous on a set E?

5. Define orthogonal system and orthogonal sequence.

6. With usual notation. Prove that $\Gamma(n+1) = n!$ for $n = 1, 2, 3, \ldots$

7. If $m^*(E) = 0$, then prove that E is measurable.

8. Prove that if f is a measurable function and $f = g$ a.e, then g is measurable.

9. Define a characteristic function and a simple function.

10. What do you mean by Lebesgue integral of f over E?

Part B (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) If f is continuous on $[a, b]$, then prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) If $f_n(x) = n^2 x(1 - x^2)^n$, where $0 \le x \le 1$, $n = 1, 2, 3\ldots$ then prove that the limit of the integral need not be equal to the integral of the limit.

Or

(b) State and prove the cauchy criterion for uniform convergence.

13. (a) Prove the theorem for pointwise convergence of Fourier series with statement.

Or

(b) If f is continuous with period 2π and if $\in > 0$, then prove that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \in$ for all real x.

14. (a) Prove that the union of two measurable sets is measurable.

Or

(b) Show that every Borel set is measurable. In particular each open set and each closed set is measurable.

15. (a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure, then prove that $\int[a\phi + b\psi] = a\int\phi + b\int\psi$.

Or

(b) If f and g are bounded measurable functions defined on a set E of finite measure, then prove that $\int_E [af + bg] = a\int_E f + b\int_E g$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

17. State and prove the stone-Weierstrass theorem.

18. Prove the Parseval's theorem with the statement.

19. Show that the outer measure of an interval is its length.

20. State and prove the Bounded convergence theorem.

S–2364

Sub. Code
22MMA2C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a partial differential equation. Give an example.

2. Define a complete integral of the equation.

3. Verify that the equation $Z = \sqrt{2x+a} + \sqrt{2y+b}$ is a complete integral of the partial differential equation $Z = \frac{1}{p} + \frac{1}{q}$.

4. Is along every characteristic strip of the equation $F(x, y, z, p, q) = 0$ the function $F(x, y, z, p, q)$ is a constant? Justify.

5. Find a complete integral of the equation $p + q = pq$.

6. Write short notes on Jacobi's method for solving partial differential equation.

7. If $u = f(x+iy) + g(x-iy)$, where the functions f and g are arbitrary, show that $\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 x}{\partial y^2} = 0$.

8. Give an example for each of the hyperbolic and parabolic type second order partial differential equations.

9. Write down the exterior Dirichlet boundary value problem for Laplace's equation.

10. State the One-dimensional diffusion equation.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Eliminate the constants a and b from the equation $ax^2 + by^2 + z^2 = 1$.

Or

(b) Form a partial differential equation, by eliminating the function f from $z = x + y + f(xy)$.

12. (a) If u is a function of x, y and z which satisfies the partial differential equation.

$(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$. Show that u contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

Or

(b) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

13. (a) Show that the equation $xp - yq = x, x^2 p + q = xz$ are compatible and find their solution.

Or

(b) Solve the equation $zpq = p + q$ using charpit's method.

14. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

Or

(b) By separating the variables, solve $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

15. (a) Prove that $r\cos\theta$ and $r^{-2}\cos\theta$ satisfy Laplace's equation, when r, θ, ϕ are spherical polar coordinates.

Or

(b) Derive the '*d*' Alembert's solution of the one-dimensional wave equation.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State the Cauchy's problem. Also write this properties.

(b) State the Sonia Kowalewski theorem.

17. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis.

18. Show that the only integral surface of the equation $2q(z-px-qy)=1+q^2$ which is circumscribed about the paraboloid $2x=y^2+z^2$ is the enveloping cylinder which touches it along its section by the plane $y+1=0$.

19. Derive the solution of the equation $\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}=0$ for the region $r \geq 0$, $z \geq 0$, satisfying the condition:

(a) $V \to 0$ and $z \to \infty$ and as $r \to \infty$.

(b) $V = f(r)$ on $z=0, r \geq 0$.

20. Determine the temperature $\theta(\rho,t)$ in the infinite cylinder $0 \leq \rho \leq a$ when the initial temperature is $\theta(\rho,0)=f(\rho)$ and the surface $\rho = a$ is maintained at zero temperature.

S–2365

Sub. Code
22MMA2C4

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a random variable.

2. Sketch the graph for $F(x)=\begin{cases} 0, & x<-1 \\ \frac{x+2}{4}, & -1 \le x < 1 \\ 1, & 1 \le x \end{cases}$.

3. Prove that $E[E(X_2/X_1)] = E(X_2)$.

4. Define covariance of X and Y.

5. Write down the Poisson Postulates.

6. Let X be the random variable for the distribution $N(\mu, \sigma^2)$. If $\Pr(X \le 60) = 0.10$ and $\Pr(X \le 90) = 0.95$, Find the values of μ and σ.

7. Let X_1, X_2, X_3 be a random sample of size 3 from a distribution that is $N(6,4)$. Determine the probability that the largest sample observation exceeds 8.

8. Write down the mean and variance of the F-distribution.

9. When we say that a sequence of random variables $X_1, X_2, \ldots,$ converges in probability to a random variable X?

10. Let X be $\chi^2(50)$. Approximate $\Pr(40 < X < 60)$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $f(x) = \frac{1}{x^2}, 1 < x < \infty$, zero elsewhere be the p.d.f. of X. If $A_1 = \{x : 1 < x < 2\}$ and $A_2 = \{x : 4 < x < 5\}$. Find $P(A_1 \cup A_2)$ and $P(A_1 \cap A_2)$.

Or

(b) Let X have the p.d.f. $f(x) = \frac{x+2}{18}, -2 < x < 4$, zero elsewhere. Find $E\left[6X - 2(X+2)^3\right]$ and $E\left[(X+2)^2\right]$.

12. (a) Let X_1 and X_2 have the joint p.d.f. $f(x_1, x_2) = 15x_1^2 x_2, 0 < x_1 < x_2 < 1$, zero elsewhere. Find each marginal p.d.f and compute $\Pr(X_1 + X_2 \le 1)$.

Or

(b) Show that the random variables X_1 and X_2 with joint p.d.f. $f(x_1, x_2) = 12x_1 x_2(1 - x_2), 0 < x_1 < 1$, $0 < x_2 < 1$, zero elsewhere, are independent.

13. (a) Compute the measures of skewness and kurtosis of a distribution which is $N(\mu, \sigma^2)$.

Or

(b) Derive the p.d.f. of Poisson distribution.

14. (a) Let X have a p.d.f. $f(x) = \frac{1}{3}, x = 1, 2, 3$ zero elsewhere. Find the p.d.f. of $Y = 2X + 1$.

Or

(b) Let X_1, X_2, X_3, X_4 be four i.i.d. random variables having the same p.d.f. $f(x) = 2x, 0 < x < 1$, zero elsewhere. Find the mean and variance of the $Y = X_1 + X_2 + X_3 + X_4$.

15. (a) Let X_n have a gamma distribution with parameter $\alpha = n$ and β, where β is not a function of n. Let $Y_n = X_{n/n}$. Find the limiting distribution of Y_n.

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Further, Let U_n converge in probability to the +ve constant C and let $\Pr(U_n < 0) = 0$ for every n. Prove that the random variable $\sqrt{u_n}$ converges in probability to $\sqrt{c}$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) From a bowel containing 5 red, 3 white and 7 blue chips, select 4 at random and without replacement. Compute the conditional probability of 1 red, 0 white and 3 blue chips, given that there are at least 3 blue chips in this sample of 4 chips.

(b) Find the moments of the distribution that has m.g.f. $M(t) = (1-t)^{-3}, t > 1$.

17. Let $f_{1/2}(x_1/x_2)= \dfrac{C_1x_1}{x_2^2}, 0 < x_1 < x_2, \quad 0 < x_2 < 1,$ zero elsewhere and $f_2(x_2) = C_2x_2^4, \quad 0 < x_2 < 1,$ zero elsewhere, the conditional p.d.f. of x_1 given $X_2 = x_2$ and the marginal p.d.f. of X_2. Find

(a) the constants C_1 and C_2 ;

(b) the joint p.d.f. X_1 and X_2 ;

(c) $\Pr\left(\frac{1}{4} < X_1 < \frac{1}{2} \Big/ X_2 = \frac{5}{8}\right)$.

18. Find the moment generating function, mean and variance of the normal distribution.

19. Derive the p.d.f. of the beta distribution.

20. State and prove the control limit theorem.

S–2366

Sub. Code
22MMA2E1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms :

   (a) Scalar cardinality ;

   (b) Fuzzy cardinality.

2. When will you say that a fuzzy sets are said to be extension principle?

3. Write down the axiomatic skeleton for fuzzy complements.

4. What is meant by pseudo - complemented distributive lattice on $\wp(x)$?

5. Define a fuzzy relation. Given an example.

6. Write down the algorithm for transitive closure $R_T(x,x)$.

7. Write down the formula for Dempster's rule for combination.

8. State the Bayesian belief measures.

9. Define an index of fuzziness.

10. Define a joint entropy.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Enumerate the following terms with an illustration for each:

(i) Support of a fuzzy set;

(ii) Height of a fuzzy set;

(iii) Convex fuzzy set;

(iv) Level set of a fuzzy set.

Or

(b) Show that all α - cuts of any fuzzy set A defined on $\mathbb{R}^n$ $(n \geq 1)$ are convex if and only if

$\mu_A(\lambda\bar{r} + (1-\lambda)\bar{s}) \geq$ min $[\mu_A(\bar{r}), \mu_A(\bar{s})]$ for all $\bar{r}, \bar{s} \in \mathbb{R}^n$ and all $\lambda \in [0,1]$.

12. (a) If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

Or

(b) Prove that

$$\lim_{w \to \infty} i_w = \lim_{w \to \infty}\left(1 - \min\left[1, \left((1-a)^w + (1-b)^w\right)^{\frac{1}{w}}\right]\right)$$

13. (a) What is meant by a sagittal diagram? Explain with suitable illustration.

Or

(b) Determine the transitive max-min closure for a fuzzy relation defined by the membership matrix.

$$\begin{bmatrix} 0.7 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}$$

14. (a) Prove that a belief measure Bel on a finite power set $\mathcal{P}(x)$ is a probability measure if and only if its basic assignment m is given by $m(\{x\}) = \text{Bel}\ (\{x\})$ and $m(A) = 0$ for all subsets for x that are not singletons.

Or

(b) Show that the function bel determined by equation Bel $(A) = \sum_{B \subseteq A} m(B)$ for any given basic assignment m is a belief measure.

15. (a) Consider two fuzzy sets, A and B defined on the set of real numbers $x = [0,4]$ by the membership grade functions $\mu_A(x) = \dfrac{1}{1+x}$ and $\mu_{\mathbf{B}}(x) = \dfrac{1}{1+x^2}$. Draw graphs for these functions and their standard classical complements.

Or

(b) Enumerate the following terms:

(i) Boltzmann entropy

(ii) Shannon cross - entropy.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Consider the fuzzy sets A, B and C defined on the interval $x = [0,10]$ of real numbers by the membership grade functions.

$$\mu_A(x) = \frac{x}{x+2}, \mu_B(x) = 2^{-x}, \mu_C(x) = \frac{1}{1+10(x-2)^2}.$$

Determine mathematical formulas and grad of the membership grade functions of each of the following:

(a) $\overline{A}, \overline{B}, \overline{C}$;

(b) $A \cup B,\ A \cup C, B \cup C$;

(c) $A \cap B, A \cap C, B \cap C$’

(d) $A \cup B \cup C$;

(e) $\overline{A \cup C}$

17. Prove that fuzzy set operations of union, intersection, and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent or distributive

18. Enumerate the following types of fuzzy relations:

(a) Reflexive

(b) Symmetric

(c) Transitive

(d) Antireflexive

(e) Antisymmetric

19. Prove : Given a consonant body of evidence $(\mathcal{F}, m)$, the associated consonant belief and plausibility measures possess the following properties.

(a) Bel $(A \cap B) = \min[Bel(A), Bel(B)]$ for all A,B $\in \mathcal{P}(x)$;

(b) Pl $(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in \mathcal{P}(x)$.

20. State and prove the Gibbs’ theorem.

S–2367

Sub. Code
22MMA2E2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

Elective — NUMERICAL METHODS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define order, convergence and asymptotic error constant.

2. State Sturm theorem.

3. What is matrix norm? Give the properties.

4. Write the condition for convergence of an iterative method.

5. What is meant by finite elements and knots?

6. What is a cubic spline?

7. Define error of approximation.

8. What is Extrapolation method?

9. What is the order of error in Simpson's formula?

10. What is the order of error in trapezoidal formula?

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $P_4(x) = 4x^4 + 2x^2 - 1 = 0$ using Sturm sequences.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use the initial approximation $p_0 = 0.5, q_0 = 0.5$.

12. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ using the iterative method, given that its approximate inverse is $B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$.

Or

(b) For the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

(i) find all the eigen values and the corresponding eigenvectors.

(ii) Verify $S^{-1} AS$ is a diagonal matrix, where S is the matrix of eigenvectors.

13. (a) Using the following values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
–1	1	–5
0	1	1
1	3	7

Estimate the values of f (–0.5) and f (0.5) using piecewise cubic Hermite interpolation.

Or

(b) Given the data

x	0	1	2	3
$f(x)$	1	2	33	244

fit quadratic splines with $M(0) = f''(0) = 0$. Hence, find an estimate of $f(2.5)$.

14. (a) A differentiation rule of the form $h f'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given. Determine the values of $\alpha_0, \alpha_1, \alpha_2$ and α_3 so that the rule is exact for a polynomial of degree 4.

Obtain an expression for the round off error in calculating $f'(x_2)$.

Or

(b) Define $S(h) = \dfrac{-y(x+2h) + 4y(x+h) - 3y(x)}{2h}$ Show that $y'(x) - S(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 \ldots.$ and state c_1.

15. (a) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using Simpson's rule.

Or

(b) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$, using trapezoidal rule.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Find all the roots of the polynomial $x^4 - x^3 + 3x^2 + x - 4 = 0$ using the Graeffe's root squaring method.

17. Find all the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain the cubic spline approximation for the function defined by the data

x	0	1	2	3
$f(x)$	1	2	33	244

with $M(0) = 0$, $M(3) = 0$. Hence, find an estimate of $f(2.5)$.

19. Derive the formulas for the first derivative of $y = f(x)$ of $O(h^2)$ using

(a) Forward difference approximations,

(b) Backward difference approximations,

(c) Central difference approximations.

20. Find the remainder of the Simpson three-eighth rule $\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f(x_0) + 3f(x_2) + f(x_3)\right]$, for equally spaced points $x_i = x_0 + ih, i = 1, 2, 3$. Use this rule to approximate the value of the integral $I = \int_0^1 \frac{dx}{1+x}$, Also find a bound on the error.

S–2369

Sub. Code
22MMA2N1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Second Semester

Mathematics

NME — DISCRETE MATHEMATICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the following terms. Give an example for each :
   (a) Conjunction
   (b) Disjunction
2. What is a well-formed formula?
3. Define an elementary product and elementary sum. Give an example for each.
4. What is meant by valid argument? Give an example.
5. If $A = \{1,2,3,4\}$ and $R = \{(1,1), (1,3), (2,3), (3,2), (3,3), (4,3)\}$, draw the diagraph of R.
6. Define an equivalence relation. Give an example.
7. Define a chain. Give an example.
8. Define a lattice homomorphism.

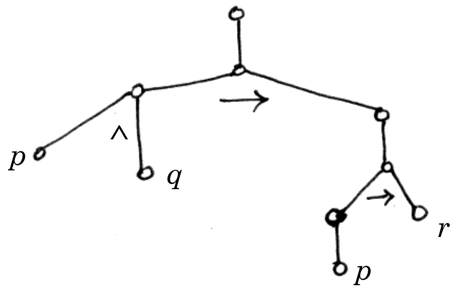
9. What is meant by atom of the lattice?

10. Define a complete sum of the n-variables in Boolean Polynomial.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the formula corresponding to the tree given in the following figure.



Or

(b) Define a tautology, verify whether $(p \vee q) \vee (p \wedge q)$ is a contradiction or tautology.

12. (a) Obtain a disjunctive normal form of $\neg (P \vee Q) \leftrightarrow (P \wedge Q)$.

Or

(b) By not using the truth table directly, find PDNF for

(i) $P \leftrightarrow Q$

(ii) $\neg P \vee Q$

13. (a) A relation R in C, the set of all complex numbers is defined as follows : $x\,R\,y \Leftrightarrow x$ is the conjugate of y. Which of the following statements are true?

(i) $4R4$ (ii) iRi

(iii) $-2R2$ (iv) $(1+i)R(1-i)$

(v) $(-1+i)R(-1-i)$

Or

(b) Let a relation R be defined on the set of all real numbers by "if x, y are real numbers, $x\,R\,y \Leftrightarrow x-y$ is a rational number". Show that the relation R is an equivalence relation.

14. (a) Prove that in any lattice $(<, \leq)$, the operations $\vee$ and $\wedge$ are isotone.

Or

(b) Show that every chain is a distributive lattice.

15. (a) In a Boolean algebra, show that

(i) $a \vee (a' \wedge b) = a \vee b$

(ii) $a \wedge (a' \vee b) = a \wedge b$

Or

(b) Find the principal disjunctive normal form of $P(x_1, x_2, x_3) = (x_2 + x_1 x_3)\left(\overline{(x_1 + x_3)x_2}\right)$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that $(\urcorner P \wedge (\urcorner Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

(b) Prove that $((P \vee Q) \wedge \urcorner(\urcorner P \wedge (\urcorner Q \vee \urcorner R))) \vee (\urcorner P \wedge \urcorner Q)$ $\vee (\urcorner P \wedge \urcorner R)$ is a tautology.

17. If $p \to q, q \to r, \urcorner(p \wedge r)$ and $p \vee r$, then prove that r.

18. (a) With the usual notations, prove that associative law for composition of relations for three sets.

(b) Let A, B and C be sets. R is a relation from A to B and S is a relation from B to C. Prove that $(SoR)^{-1} = R^{-1} o S^{-1}$.

19. (a) Prove that every chain is a lattice.

(b) Let $(<, \leq)$ be a lattice and for any $a, b \in L$. Prove the following are equivalent :

(i) $a \leq b$ (ii) $a \vee b = b$

(iii) $a \wedge b = a$

20. (a) Define a Boolean algebra with an example.

(b) Is the lattice of divisors of 32 a Boolean algebra? Justify.

(c) Express the polynomial $P(x_1, x_2, x_3) = x_1 \vee x_2$ is an equivalent sum-of-products canonical form in three variables x_1, x_2 and x_3.

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S–2370

Sub. Code
22MMA3C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the complex form of the Cauchy-Riemann equations.

2. Distinguish between translation, rotation and inversion.

3. Compute $\int_{\gamma} x dz$ where γ is the directed line segment from 0 to $1+i$.

4. State the fundamental theorem of algebra.

5. Define a meromorphic function.

6. Define an essential singularity. Give an example.

7. Find the residue for $\frac{1}{\sin z}$ at $z=0$.

8. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$?

9. State the Hurwitz theorem.

10. What is meant by entire function? Give an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Expand $\dfrac{2z+3}{z+1}$ in powers of $z-1$. What is the radius of convergence?

Or

(b) Define a linear transformation. Also prove that the reflection $z \to \bar{z}$ is not a linear transformation.

12. (a) Prove that the line integral $\int_\gamma p\,dx + q\,dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\dfrac{\partial U}{\partial x} = p$, $\dfrac{\partial U}{\partial y} = q$.

Or

(b) State and prove the Morera's theorem.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the argument principle theorem.

Or

(b) Evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

15. (a) Prove the following:

(i) $\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}$;

(ii) $(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots=\frac{1}{1-z}$ if $|z|<1$.

Or

(b) Derive the Poisson – Jensen formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Define the cross ratio. Also prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

17. State and prove the Cauchy's representation formula. Also deduce that $f^{(n)}(z)=\frac{n!}{2\pi i}\int\frac{d(\zeta)d\xi}{(\xi-z)^{n+1}}$.

18. State and probe the Schwarz lemma.

19. (a) State and prove the residue theorem.

(b) State and prove the Rouche's theorem.

20. Obtain the Laurent expansion $\sum_{n=-\infty}^{\infty} An(z-a)^n$ for the function $f(z)$ analytic in $R_1<|z-a|<R_2$.

S–2371

Sub. Code
22MMA3C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

TOPOLOGY — I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the standard topology. Give an example.

2. Is the subset $[a, b]$ of $\mathbb{R}$ closed? Justify your answer.

3. Define a continuous function.

4. What is meant by the square metric?

5. Define the term linear continuum.

6. Define the following terms:

   (a) Locally connected;

   (b) Locally path connected.

7. Is the interval (0, 1) compact? Justify your answer.

8. State the Lebesgue number lemma.

9. Define a first-countable.

10. Whether the space $\mathbb{R}_K$ is regular or not? Justify your answer.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

Or

(b) Define the product topology with an example. Also prove that the collection $S\left\{\Pi_1^{-1}(U) / U \text{ open in } X\right\}$ U $\left\{\Pi_2^{-1}(V) / V \text{ open in } Y\right\}$ is a sub basis for the product topology on $X \times Y$.

12. (a) Let $\{X_\alpha\}$ be an indexed family of spaces and let $A_\alpha \subset X_\alpha$ for each α. If ΠX_α is given either the product or the box topology, then prove that $\Pi \overline{A_\alpha} = \overline{\pi_{A_\alpha}}$.

Or

(b) State and prove uniform limit theorem.

13. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Prove that a space X is locally path connected if and only if for every open set U of X, each path component of U is open in X.

14. (a) State and prove Extreme value theorem.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Suppose that X has a countable basis. Prove that every open covering of X contains a countable subcollection covering X.

Or

(b) Prove that a subspace of a Hausdorff space is Hausdorff.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X.

(b) Let X be a topological space. Prove the following conditions hold:

(i) ϕ and X are closed.

(ii) Arbitrary intersections of closed sets are closed.

(iii) Finite unions of closed sets are closed.

17. Let X and Y be topological spaces and let $f : X \to Y$. Prove the following are equivalent:

(a) F is continuous.

(b) For every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$.

(c) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

(d) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

18. (a) State and prove Intermediate value theorem.

(b) What are the components and path components of $\mathbb{R}_\ell$? What are the continuous map $f: \mathbb{R} \to \mathbb{R}_\ell$?

19. Show that the product of finitely many compact spaces is compact.

20. State and prove the Urysohn Lemma.

S–2372

Sub. Code
22MMA3C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours　　　　Maximum : 75 Marks

Part A　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. Define a regular graph with an example.

2. What is meant by the girth of graph G?

3. Define a edge connectivity. Give an example.

4. Define block of a graph with an example.

5. State the Berge theorem.

6. Determine the edge chromatic number of a Petersen graph.

7. Define the following terms with an example for each :

   (a) Independent set ;

   (b) Maximum independent set.

8. State the Brook's theorem.

9. Embed K_5 on the torus.

10. Define the dual of a plane graph. Give an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Prove that in any graph, the number of vertices of odd degree is even. Also prove that $\delta \leq \frac{2\varepsilon}{v} \leq \Delta$, by using usual notations.

Or

(b) If G is a tree, then prove that $\varepsilon = \gamma - 1$.

12. (a) If G is simple and $\delta \geq (\gamma + k - 2)/2$, then prove that G is k-connected. Also prove that $k = k'$ if G is simple and 3-regular.

Or

(b) Prove that a connected graph has an Euler trial if and only if it has atmost two vertices of odd degree.

13. (a) If G is a k-regular bipartite graph with $k > 0$, then prove that G has a perfect matching.

Or

(b) If G is bipartite, then prove that $\chi' = \Delta$.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G. Also prove that $\alpha + \beta = \gamma$.

Or

(b) Define a critical graph with an example. Also prove that every critical graph is a block.

15. (a) State and prove the Euler's formula for a connected plane graph.

Or

(b) If two bridges overlap, then prove that they are skew or else they are equivalent 3-bridges.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Show that every simple graph on n vertices is isomorphic to a subgraph of K_n.

    (b) If $\delta \geq 2$, then prove that G contains a cycle.

17. State and prove the Chvatal theorem.

18. If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. Let G be a k-critical graph with a 2-vertex cut $\{u, v\}$. Prove the following :

    (a) $G = G_1 \cup G_2$, where G_i is a $\{u, v\}$ - component of type $i(i = 1,\ 2)$ and

    (b) Both $G_1 + uv$ and $G_2 . uv$ are k-critical (where $G_2.uv$ denote the graph obtained from G_2 by identifying u and v).

20. Show that every planar graph is 5-vertex colourable.

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S–2373

Sub. Code
22MMA3C4

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

MECHANICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define configuration space.

2. What is meant by the potential energy?

3. Write down the standard form of Lagrange's equation for a non holonomic system.

4. When do you say that a conservative system is a natural system?

5. Write down the multiplier rule for finding the stationary value of an integral.

6. State the Hamilton's canonical equations of motion for a holonomic system.

7. Define "Pfaffian differential forms".

8. Write down the modified Hamilton-Jacobi equation.

9. When will you say that the transformation is said to be Mathieu transformation?

10. Define the Poisson bracket.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove D'Alembert's principle.

Or

(b) State and prove Konig's theorem.

12. (a) A particle of mass m is connected by a massless spring of stiffness k and unstressed length r_0 to a point p which is moving along a circular path of radius a at a uniform angular rate w. Assuming that the particle moves without friction on a horizontal plane, find the differential equations of motion.

Or

(b) Discuss the Kepler problem, using ignorable coordinates.

13. (a) Find the stationary values of the function $f = z$, subject to the conditions $x^2 + y^2 + z^2 = 4$ and $xy = 1$.

Or

(b) Find the equations of motion for a charged particle in an electromagnetic field, using Hamiltonian H.

14. (a) Discuss the Hamilton's principal function.

Or

(b) Using Hamilton-Jacobi method discuss the problem of a mass-spring system.

15. (a) Define a homogeneous canonical transformation and analyse the generating functions associated with such a transformation.

Or

(b) Show that the transformation $Q = \log\left(\frac{\sin p}{p}\right)$, $p = q \cot p$ is canonical.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. A particle of mass m is suspended by a massless wire of length $\vec{r} = a + b\cos\omega t$ $(a > b > 0)$ to form a spherical pendulum. Find the equations of motion.

17. Derive the Lagrange's equation in terms of Routhian function in the form $\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_i}\right) - \frac{\partial R}{\partial q_i} = 0$, $(i = k+1, \ldots\ldots, n)$.

18. Using Jacobi's form of the principle of least action, obtain the orbit for the Kepler problem in the form $r = \frac{C^2 / \mu m^2}{\sqrt{1 + 2hC^2 / \mu^2 m^3 \cos\theta}}$, with an eccentricity $e = \sqrt{1 + 2hC^2 / \mu^2 m^3}$.

19. State and prove the Jacobi's theorem.

20. Define Lagrange bracket $[u, v]$ and prove that $[q_j, q_k] = [p_j, p_k] = 0$ while $[q_j, p_k] = \delta_{jk}$.

S–2374

Sub. Code
22MMA3E1

M.Sc. DEGREE EXAMINATION, APRIL 2024.

Third Semester

Mathematics

Elective – ADVANCED STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Let the observed value of the mean $\overline{x}$ of a random sample of size 40 from a distribution $N(\mu, 10)$ be 7.164. Find a 95% confidence interval for μ.

2. Define parameter space.

3. Define sufficient statistic for θ.

4. State Rao-Blackwell theorem.

5. Let x have a gamma distribution with $\alpha = 4$ and $\beta = \theta > 0$. Find the Fisher information $I(\theta)$.

6. Define efficiency of statistics.

7. What is mean by monotone likelihood in the statistics?

8. Write down the likelihood ratio test principle.

9. Define central chi-square variable.

10. Why we use the quadratic form in statistic?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let $x_1, x_2, \ldots, x_{25}$ be a random sample from $N(\mu, 4)$ with mean $\bar{x} = 76.1$. To test $H_0 : \mu = 77$ against the one-sided alternative hypothesis $H_1 : \mu < 77$.

Or

(b) Let x have a p.d.f of the form $f(x:\theta) = \theta x^{\theta-1}$, $0 < x < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$, use a random sample X_1, X_2 of size $n = 2$ and the critical region to be $C = \left\{(x_1, x_2); \frac{3}{4} \le x_1 x_2\right\}$. Find the power function of the test.

12. (a) If $x_1, x_2, \ldots, x_n$ is a random sample from a poisson distribution with mean $\theta > 0$. Prove that the family $\{g_1(y_1; \theta); \theta > 0\}$ is complete where $y_1 = \sum_{i=1}^{n} x_i$.

Or

(b) If x_1, x_2 is a random sample of size 2 from a distribution having p.d.f $f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $0 < x < \infty$, $0 < \theta < \infty$, zero elsewhere, Find the joint p.d.f of the sufficient statistic $y_1 = x_1 + x_2$ for θ and $y_2 = x_2$. Show that y_2 is an unbiased estimator of θ with variance θ^2.

13. (a) Given the p.d.f $f(x;\theta) = \dfrac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty$, $-\infty < \theta < \infty$. Show that the Rao-Cramer lower bound is $\dfrac{2}{n}$, where n is the size of a random sample from this Cauchy distribution.

Or

(b) Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\theta, \sigma^2)$ where $-\infty < \theta < +\infty, \sigma^2 > 0$. Prove that the sample mean $\bar{x}$ is an efficient estimator of θ.

14. (a) Let $x_1, x_2, \ldots, x_n$ denote a random sample from a distribution $N(0, \theta)$, where θ is unknown positive number. Show that there is a uniformaly most powerful test for testing $H_0 : \theta = \theta'$ against $H_1 : \theta > \theta'$ where θ' is a fixed the numbers.

Or

(b) If $x_1, x_2, \ldots, x_n$ is a random sample from a beta distribution with parameters $\alpha = \beta = \theta > 0$, find a best critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.

15. (a) Explain the test of equality of several means.

Or

(b) Consider the variance s^2 of the random sample of size $n = ab$. Verify that $Q = Q_3 + Q_4$ and Q_3 / σ^2 has a chi-square distribution with $b(a-1)$ degrees of freedom.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. The distribution function of Pareto distribution is given by $f(x;\theta_1,\theta_2)=1-\left(\frac{\theta_1}{x}\right)^{\theta_2}$, $\theta_1 \le x$, zero elsewhere, where $\theta_1 > 0$ and $\theta_2 > 0$. If $x_1, x_2, \ldots, x_n$ is a random sample from this distribution, find the maximum likelihood estimator of θ_1 and θ_2.

17. Let y_1, y_2, y_3 be the order statistics of a random sample of size 3 from the distribution with p.d.f $f(x;\theta)=\frac{1}{\theta}$, $0 < x < \theta$, $0 < \theta < \infty$, zero elsewhere. Show that $4y_1$, $2y_2$, $\frac{4}{3}y_3$ are unbiased estimators of θ and compare their variances.

18. State and prove Rao-Cramer inequality.

19. State and prove Neymann-Pearson theorem.

20. The following are observations associated with independent random samples from three normal distribution having equal variances and respective means μ_1, μ_2, μ_3.

I	II	III
0.5	2.1	3.0
1.3	3.3	5.1
–1.0	0.0	1.9
1.8	2.3	2.4
	2.5	4.2
		4.1

Compute the F-statistic that is used to test $H_0 : \mu_1 = \mu_2 = \mu_3$.

S–2376

Sub. Code
22MMA3E3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

Elective – AUTOMATA THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Draw a block diagram of a finite automation.

2. Define a transition system accepts a string w in Σ^*.

3. Let $G = (\{s\}, \{a\}, \{S \to Ss\}, S)$, find the language generated by G.

4. State the relations between type 0, type 1, type 2, type 3 languages.

5. When will you say that a set is said to be recursive?

6. What is meant by transpose set?

7. Define a regular set. Give an example.

8. List any two applications of pumping lemma.

9. Define the following terms :

(a) Leftmost derivation;

(b) Rightmost derivation.

10. Define the Chomsky normal form.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Consider the finite state machine whose transition function δ is given in the following table :

State	Input 0	Input 1
$\rightarrow \circled{q_0}$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Take $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0,1\}$, $F = \{q_0\}$. Give the entire sequence of states for the input string 110101.

Or

(b) Construct a non-deterministic finite automation accepting $\{ab, ba\}$.

12. (a) Find the highest type number which can be applied to the following productions :

(i) $S \rightarrow Aa, A \rightarrow c \mid Ba, B \rightarrow abc$

(ii) $S \rightarrow ASB \mid d,\ A \rightarrow aA$.

(iii) $S \rightarrow aS \mid ab$

Or

(b) Construct a grammar G generating $\{xx \mid x \in \{a,b\}^*\}$.

13. (a) Consider the grammar G given by $S \to OSA, 2, S \to 012$, $2A_1 \to A_1 2$, $1A_1 \to 11$. Test whether (i) 00112$\in L(G)$; (ii) 001122 $\in L(G)$.

Or

(b) Show that the class L_{csl} is closed under the transpose operation.

14. (a) Construct a DFA with reduced states equivalent to the regular expression 10 + (0 + 11)0 * 1.

Or

(b) If L is regular then prove that L^T is also regular.

15. (a) When will you say that a context – free grammar G is said to be ambiguous? Also if G is the grammar $S \to SbS \mid a$, then prove that G is ambiguous.

Or

(b) Find a grammar in CNF equivalent to $S \to aAbB$, $A \to aA \mid a$, $B \to bB \mid b$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) Write down the characteristics of automation. Also prove that for any transition function δ and for any two input strings x and y, $\delta(q, xy) = \delta(\delta(q, x), y)$.

(b) If $\delta(q, x) = \delta(q, y)$, then prove that $\delta(q, xz) = \delta(q, yz)$ for all strings z in Σ^+.

17. Construct a grammar G generating $\{a^n b^n c^n / n \geq 1\}$.

18. Show that a context sensitive language is recursive.

19. State and prove the Arden's theorem.

20. Let $G = (V_N, \Sigma, P, S)$ be a CFG. Prove that $S \overset{*}{\Rightarrow} \alpha$ if and only if there is a derivation tree for G with yield α.

S–2377

Sub. Code
22MMA3N1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Third Semester

Mathematics

NME – STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define the following terms:
   (a) Skewness; (b) Kurtosis.

2. What is meant by the residuals?

3. Define correlation. Give an example.

4. Write down the spearman's formula for rank correlation.

5. Given $(A) = 30$; $(B) = 25$; $(\alpha) = 30$.

   Find (a) N; (b) (β).

6. When will you say that a set of class frequencies is said to consistent?

7. Write down the formula for geometric mean index number.

8. Define the time series. Give an example.

9. When will you say that the events $A_1, A_2, \ldots\ldots, A_n$ are said to be mutually independent?

10. State multiplication theorem for probabilities.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Calculate the Karl Pearson's coefficient of skewness for the following data:

Wages in Rs. :	10	11	12	13	14	15
Frequency :	2	4	10	8	5	1

Or

(b) Fit a straight line to the following data:

x :	0	1	2	3	4
y :	1	1.8	3.3	4.5	6.3

12. (a) With the usual notations, prove that $-1 \leq \gamma \leq 1$.

Or

(b) Find the rank correlation coefficient between the height in c.m. and weight in kg of 6 soldiers in Indian Army.

Height :	165	167	166	170	169	172
Weight :	61	60	63.5	63	61.5	64

13. (a) Given that $(A)=(\alpha)=(B)=(\beta)=\frac{N}{2}$. Prove the following:

(i) $(AB)=(\alpha\beta)$;

(ii) $(A\beta)=(\alpha B)$.

Or

(b) If $\frac{(A)}{N}=x$, $\frac{(B)}{N}=2x$, $\frac{(C)}{N}=3x$ and $\frac{(AB)}{N}=\frac{(AC)}{N}=\frac{(BC)}{N}=y$, then prove that neither x nor y can exceed $\frac{1}{4}$.

14. (a) Narrate the following terms in time series:

(i) Short term fluctuations; (ii) Seasonal variation;

(iii) Cyclical variation.

Or

(b) An enquiry into the budgets of the middle class families in a city in India gave the following information.

	Food	Rent	Clothing	Fuel	Misc
Weights	35%	15%	20%	10%	20%
Prices 1991	1500	300	450	70	500
Prices 1992	1650	325	500	90	550

What change in cost of living index of 1992 as compared with that of 1991 are seen?

15. (a) State and prove the Baye's theorem.

Or

(b) The chances that 4 students A, B, C, D solve a problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{4}$ respectively. If all of them try to solve the problem, what is the probability that the problem is solved?

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Fit a curve of the form $y = ab^x$ to the following data :

Year (x) :	1951	1952	1953	1954	1955	1956	1957
Production in tons (y) :	201	263	314	395	427	504	612

17. The following data relate to the marks of 10 students in the internal test and the university examination for the maximum of 50 in each.

Internal marks	25	28	30	32	35	36	38	39	42	45
University marks	20	26	29	30	25	18	26	35	35	46

(a) Obtain the two regression equations and determine.

(b) The most likely internal mark for the university mark of 25.

(c) The most likely university mark for the internal mark of 30.

18. (a) Define the Yule's coefficient of association Q and coefficient of colligation Y. Also prove Yule's coefficient Q and the coefficient of colligation Y is related by the relation $Q = \frac{2Y}{1+Y^2}$.

(b) Investigate from the following data between inoculation against small pose and preventation from attack.

	Attacked	Not attacked	Total
Inoculated	25	220	245
Not inoculated	90	160	250
Total	115	380	495

19. Use the method of least squares and fit a straight line trend to the following data given from 1982 to 1992. Hence estimate the trend value for 1993.

Year	1982	83	84	85	86	87	88	89	90	91	1992
Production in Quintals	45	46	44	47	42	41	39	42	45	40	48

20. The contents of 3 urns are

Urn I : 1 white 3 red 2 black balls

Urn II : 3 white 1 red 1 black balls

Urn III : 3 white 3 red 3 black balls

Two balls are chosen from a randomly selected urn. If the balls are 1 white and 1 red ball what is the probability that they come from urn II?

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S–2378

Sub. Code
22MMA4C1

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. State the Jensen's inequality.

2. Define Sup norm on a function space.

3. What do you mean by support functional?

4. Define a convex body.

5. Define a Bomach space with an example.

6. Define the standard Schauder basis.

7. When will you say that a map F is said to be closed?

8. Is a linear map open? Justify your answer.

9. Define an orthonormal basis.

10. State projection theorem.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Define a normed space with an example. Let Y be a subspace of a normed space X. Prove that Y and its closure $\overline{Y}$ are normed spaces with the induces norm.

Or

(b) Let X and Y be normed spaces and $F : X \to Y$ be a linear map such that the range $R(F)$ of F is finite dimensional. Prove that F is continuous if and only if the zero space $Z\,(F)$ of F is closed in X.

12. (a) Let X be a normed space over K and f be a non zero linear functional on X. If E is an open subset of X, then prove that $f\,(E)$ is an open subset of K.

Or

(b) Let Y be a subspace of X and $a \in X$ but $a \notin \overline{Y}$. Prove that there is some $f \in X'$ such that $f_{l\,y} = 0$, $f\,(a) = dist\,(a, \overline{Y})$ and $\|F\| = 1$. Consequently, $x \in \overline{Y}$ if and only if $x \in X$ and $f\,(x) = 0$ whenever $f \in X$ and $f_{l\,y} = 0$.

13. (a) Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

Or

(b) State and prove uniform boundedness principle.

14. (a) Let X and Y be Banach spaces and $F : X \to Y$ be a closed linear map. Prove that F is continuous.

Or

(b) Let X and Y be normed spaces. If Z is a closed subspace of X, then prove that the quotient map Q from X to $\frac{X}{Z}$ is continuous and open.

15. (a) Derive the Bessel's inequality.

Or

(b) State and prove the Parseval formula.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Let X and Y be normed spaces and $F : X \to Y$ be a linear map. Prove the following conditions are equivalent:

(a) F is bounded on $\overline{U}(o,r)$ for some $r > 0$.

(b) F is continuous at 0.

(c) F is continuous on X.

(d) F is uniformly continuous on X.

(e) $\|F(x)\| \leq \alpha \|x\|$ for all $x \in X$ and some $\alpha > 0$.

(f) The zero space $Z(F)$ of F is closed in X and the linear map $\widetilde{F} : \frac{X}{Z(F)} \to Y$ defined by $\widetilde{F}(x + Z(F)) = F(x), x \in X$, is continuous.

17. State and prove the Hahn-Banach separation theorem.

18. (a) State and prove the Banach-Steinhaus theorem.

    (b) State and prove Resonance theorem.

19. State and prove open mapping theorem.

20. State and prove the Riesz representation theorem.

S–2379

Sub. Code
22MMA4C2

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

TOPOLOGY–II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define the one-point Compactification.

2. Is the rationals Q locally compact? Justify your answer.

3. What is meant by completely regular space?

4. Define the Stone-Cech Compactification.

5. Define the following terms :

   (a) Open refinement

   (b) Closed refinement.

6. Define a G_s –set. Give an example.

7. When will you say that the metric space is said to be complete?

8. Define an equicontinuous.

9. Define the evaluation map.

10. Is the irrationals Baire space? Justify your answer.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Let X be a Housdorff space. Prove that x is locally compact if and only if given x in x, and given neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Let x be a space and let $\mathfrak{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Prove that any finite intersection of elements of $\mathfrak{D}$ is an element of $\mathfrak{D}$.

12. (a) Prove that a subspace of a completely regular space is completely regular.

Or

(b) If x is completely regular and non compact, then prove that $\beta(x)$ is not metrizable.

13. (a) Let A be a locally finite collection of subsets of X. Prove that the collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Let x be normal and let A be a closed G_s set in X. Prove that there is a continuous function $f : x \to [0,1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Let x be the product space $x = \pi x_{\alpha}$ and x_n be a sequence of points of x. Prove that $x_n \to x$ if and only if $\pi_{\alpha}(x_n) \to \pi_{\alpha}(x)$ for each α.

Or

(b) Let x be a compactly generated space and let (y, d) be a metric space. Prove that $\mathcal{C}(x,y)$ is closed in y^x in the topology of compact convergence.

15. (a) If Y is locally compact Hausdorff, then prove that composition of maps $\mathcal{C}(x,y) \times \mathcal{C}(y,z) \to \mathcal{C}(x,z)$ is continuous.

Or

(b) Show that every locally compact Hausdorff space is a Baire space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an arbitrary product of compact spaces is compact in the product topology.

17. Let x be a completely regular space. Prove that there exists a compactifiation Y of X having the property that every bounded continuous map $F : x \to \mathbb{R}$ extends uniquely to a continuous map of Y into $\mathbb{R}$.

18. Let X be a regular space with a basis $\mathcal{B}$ that is countably locally finite. Prove that X is normal and every closed set in x is a G_s set in x.

19. Let I = [0,1]. Prove that there exists a continuous map $f : I \to I^2$ whose image fills up the entire square I^2.

20. State and prove the Baire category theorem.

S–2380

Sub. Code
22MMA4C3

M.Sc. DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a connected network.

2. Define a cut and the cut capacity of a network.

3. Define the following terms:

   (a) Forward Pass

   (b) Backward Pass

4. Write down the red-flagging rule.

5. Define reorder point. Give an example.

6. Define the following terms:

   (a) Holding cost

   (b) Shortage cost

7. Identify the customer and the server for the following situations:

   (a) Planes arriving at an airport

   (b) Check-out operation in a supermarket

8. What is forgetfulness property?

9. Write down the balance equation in queuing system.

10. Find p_0 of the model (M/M/1) : (GD/N/∞).

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the minimal spanning tree algorithm.

Or

   (b) Explain three-Jug puzzle with a suitable diagram.

12. (a) Construct the project network comprised of activities A to L with the following precedence relationships:

   (i) A, B and C, the first activities of the project, can be executed concurrently.

   (ii) A and B precede D.

   (iii) B precedes E, F and H.

   (iv) F and C precede G.

   (v) E and H precede I and J.

   (vi) C, D, F and J precede K.

   (vii) K precedes L.

   (viii) I, G and L are the terminal activities of the project

Or

   (b) Write down the PERT procedure in all steps

13. (a) A company stocks an item that is consumed at the rate of 50 units per day. It costs the company \$20 each time an order is placed. An inventory unit held in stock for a week will cost \$.35.

(i) Determine the optimum inventory policy, assuming a lead time of 1 week.

(ii) Determine the optimum number of orders per year (Based on 365 days per year)

Or

(b) Find the optimum order quantity for a product for which the price break are as follows:

Quantity	Unit Cost (Rs.)
$0 \le y_1 < 500$	10.00
$500 \le y_2$	9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs.350.00.

14. (a) Explain the role of exponential distribution.

Or

(b) In a bank operation, the arrival rate is 2 customers per minute. Determine the following:

(i) The average number of arrivals during 5 minutes.

(ii) The probability that no arrivals will occur during the next 0.5 minute.

(iii) The probability that at least one arrival will occur during the next 0.5 minute.

(iv) The probability that the time between two successive arrivals is at least 3 minutes.

15. (a) Explain steady-state measures of performance.

Or

(b) Arrival rate of telephone calls at a telephone booth is according to Poisson distribution, with an average time of a minutes between two consecutive arrivals. The length of a telephone call is assumed to be exponentially distributed with mean 3 minutes.

(i) Determine the probability that a person arriving at the booth will have to wait.

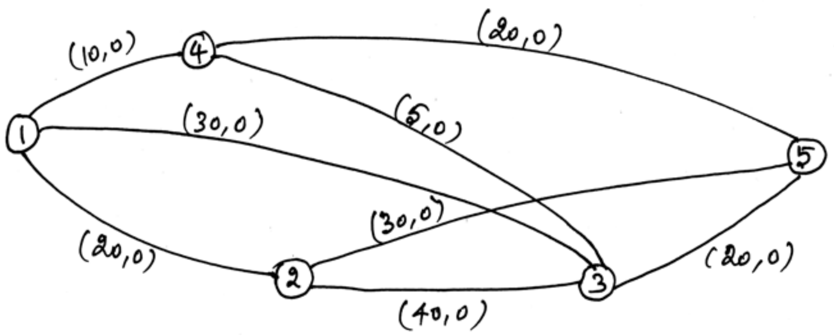
(ii) Find the average queue length that forms from time to time.

(iii) Find the fraction of a day that the phone will be in use.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Solve the following network by maximal flow method:



17. A small project is composed of activities whose time estimates are listed in the table below : Activities are identified by their beginning (i) and ending (j) node numbers.

Activity	Estimated Duration (Weeks)		
i – j	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(a) Draw the project network.

(b) Find the expected duration and variance for each activity. What is the expected project length?

(c) Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed?

(i) At least 4 weeks earlier than expected?

(ii) No more than 4 weeks later than expected time?

(d) If the project due is 19 in weeks, what is the probability of meeting the due date? Given :

Z	0.5	0.67	1.00	1.33	2.00
p	0.1915	0.2486	0.3413	0.4082	0.4772

18. Neon lights on the U of A compus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs \$100 to initiate a purchase order. A neon light kept in storage is estimated to cost about \$0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

19. The florist section in a grocery store stocks 18 dozen roses at the beginning of each week. On the average, the florist sells 3 dozens a day (one dozen at a time), but the actual demand follows a poisson distribution. Whenever the stock level reaches 5 dozens, a new order of 18 new dozens is placed for delivery at the beginning of the following week. Because of the nature of the item, all roses left at the end of the week are disposed of. Determine the following:

 (a) The probability of placing an order in any one day of the week.

 (b) The average number of dozen roses that will be discarded at the end of the week.

20. B and K Groceries operates with three check out counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in store.

No. of customers in store	No. of customers in operation
1 to 3	1
4 to 6	2
More than 6	3

Customer arrive in the counters area according to a poisson distribution with a mean rate of 10 customers per hour. The average check-out time per customer is exponential with mean 12 minutes. Determine the steady-state probability p_n of n customers in the check-out area.